Georg ZOTTI: *Universal Astrolabes*

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The planispheric astrolabe is very well suited for the rapid and illustrative solution of astronomical problems, but only for the geographic latitude of the currently inserted *tympan*. Since the construction of tympan disks was elaborate and a multitude of such disks were expensive and heavy, there were efforts to create an instrument usable for all latitudes. Two such models were able to establish themselves; a third did not see wide dissemination:

Saphæa Arzachelis (Fig. 8.1) The most widespread universal astrolabe is based on the works of ALI BEN KHALAF (*Lamina Universal*) and IBN AZ-ARQUELLA (*Saphæa Arzachelis*) (both Toledo, 11th century). RAINER GEMMA FRISIUS (1508–1555) described this design in *de astrolabo catholico liber*, but the Saphæa was already well known in Europe at that time [Zinner 1967, p.146]. Particularly famous were the universal astrolabes from the workshop of his nephews in Leuven, the ARSENIUS brothers [Greenwich 1976/1989].

de Rojas (Fig. 8.2) JUAN DE ROJAS SARMIETO, a student of GEMMA, described in 1551 an instrument that was easier to use. Instead of using stereographic projection, the celestial sphere is depicted in orthographic projection; that is, the viewpoint is placed at infinity. As a result, the circles of latitude are represented as straight lines, and the circles of longitude become elliptical arcs. The instrument is indeed easier to use, but coordinates near the edge can only be read imprecisely.

de la Hire (Fig. 8.3) PHILIPPE DE LA HIRE (1640–1718) developed a projection that aimed to minimize distortions as much as possible. However, due to the elliptical arcs, it was quite complicated in construction and no easier to use than the Saphæa. Moreover, the design was introduced during the decline of astrolabe craftsmanship, so only a few were built. Apparently, only one cardboard example survives worldwide [Greenwich 1976/1989].

All three models share the characteristic that the projection does not have a celestial pole at the center and the equatorial plane or a plane parallel to it at the opposite pole as the projection plane. Instead, the projection plane used is the plane of the solstitial colure, that is, the great circle passing through the poles and the solstice points ♋ and ♑. The projection thus shows an equatorial grid from the side, but even represents both hemispheres, depending on whether one looks at the sphere from the outside or inside. Therefore, both the vernal equinox point ♈ and the autumnal equinox point ♎ are located in the center of the projection. By silent convention, the vernal equinox point always lies on the side of the

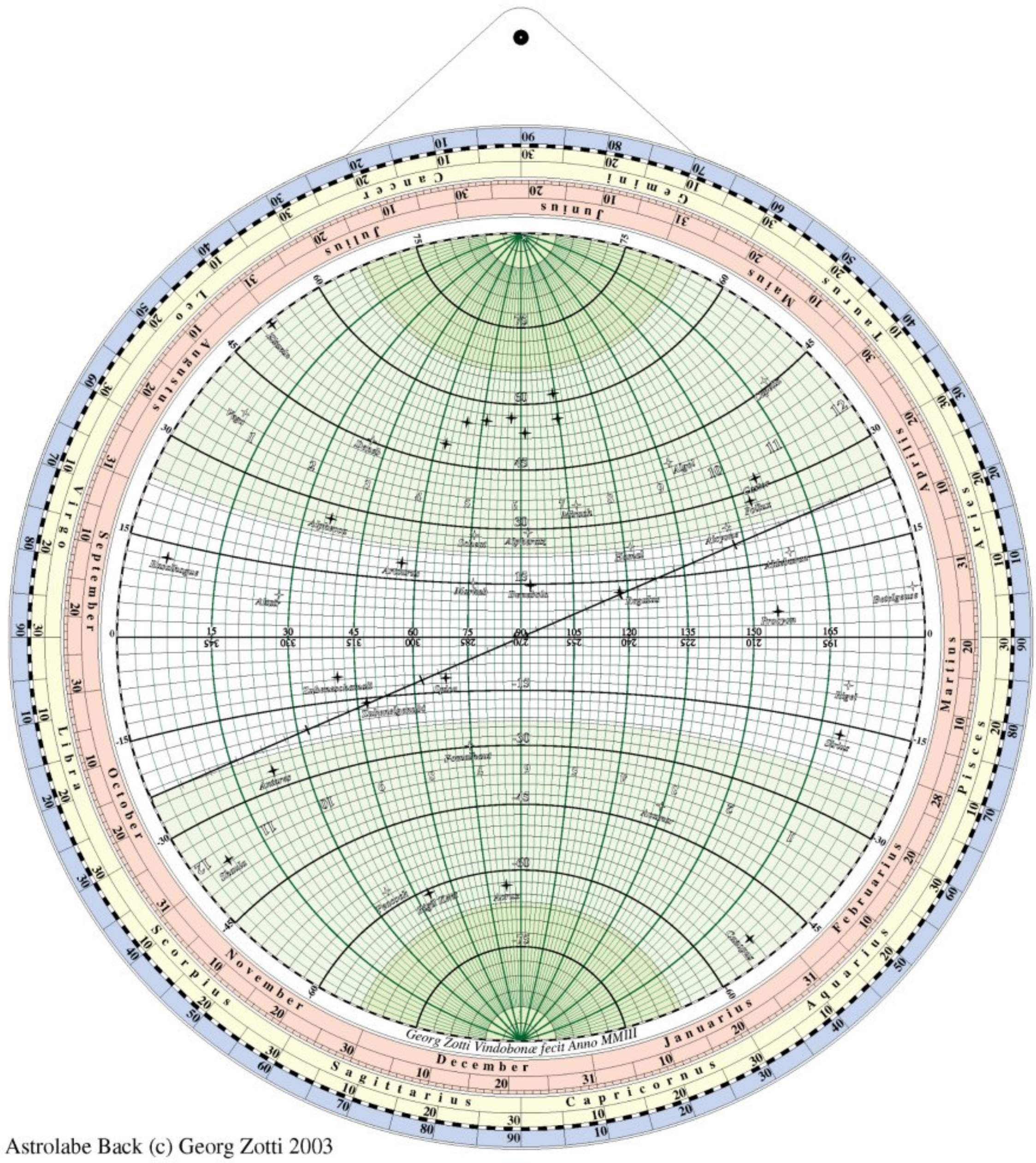

Figure 8.1: Universal Astrolabe: Saphæa Arzachelis after GEMMA FRISIUS.

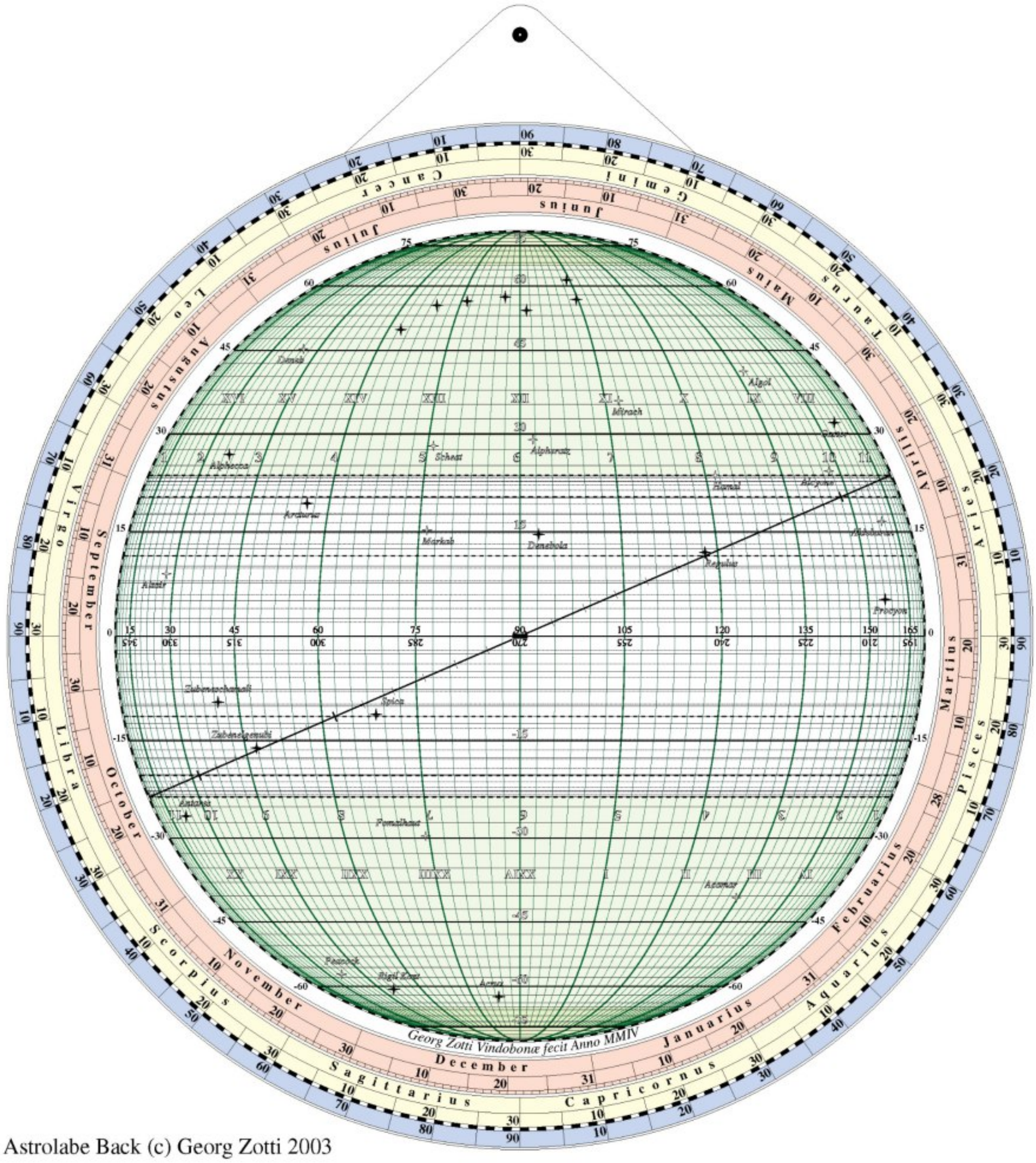

Figure 8.2: Universal Astrolabe after JUAN DE ROJAS.

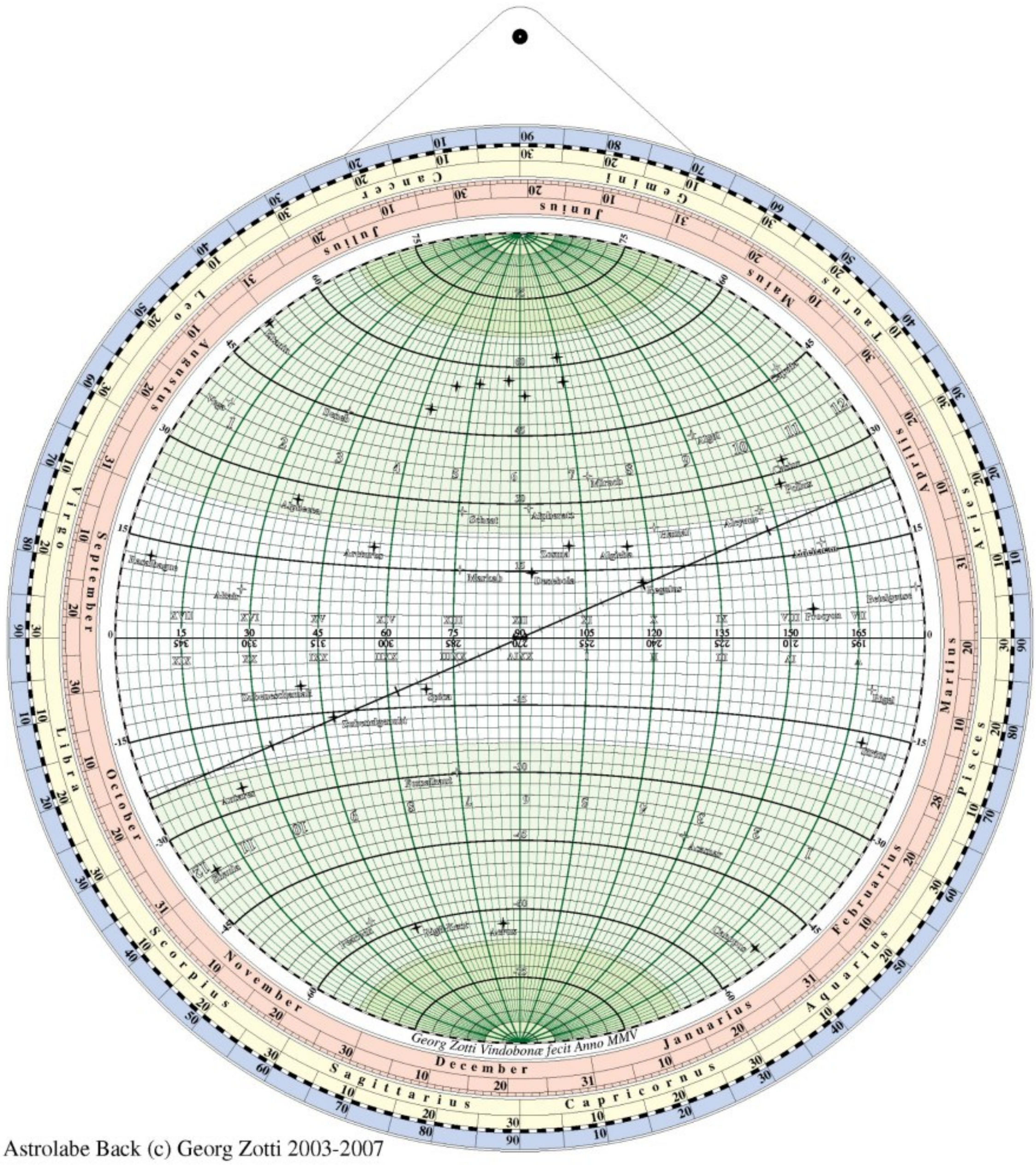

Figure 8.3: Universal Astrolabe after DE LA HIRE

user, so that the summer solstice point ♋ always lies above to the right, the winter solstice point ♑ always below to the left of the equatorial diameter, and the ecliptic extends as a slanting but straight diameter between these points.

The lengths of the meridian circles thus increase from the center (0°) to the right (90°), back through the center (180°) to the left (270°), and again to the center (360°). The center therefore corresponds to either 0° or 180° longitude. Typically, the grid is interpreted as a representation of equatorial coordinates, in which some stars are often plotted. For better distinction, stars with right ascension from 6h to 18h are, for example, fully engraved, while stars with right ascension from 0h to 6h and from 18h to 24h are engraved only as outline lines.

The same grid can be interpreted as a representation of the hour angle *H*. South (more generally: the meridian of the upper culmination) is always on the right, and since in this view problems of time determination with the Sun are usually addressed, the labeling is done directly in solar hours (2 x 12). While our current hour angle counting has zero at culmination, here it is 12, which must be considered as an additional hurdle in some tasks (Sections 8.1.12, 8.1.13). To be safe, we write $\widehat{H} = (H + 12) \bmod 24$.

In the models presented here, the ecliptic carries 5° and stronger 30° divisions. The labeling of the ecliptic is carried by the *Regula*. The thick lines are 15° apart, i.e., 1 hour in right ascension, the thin lines are 2.5°.

This instrument was usually built either as the backside of a planispheric astrolabe or combined with one of the other backsides (e.g., calendar with shadow square); in any case, the sighting devices had to be accommodated on the other side.

New Components

Above this grid is again mounted a rotating pointer: this *Regula* usually carries a continuous ecliptic scale and is, in the designs here, bounded on the other side by the -18° twilight line (*Linea auroræ*). Furthermore, since REGIOMONTANUS [Zinner 1967, p.147], it carries on the *Saphæa* a small hinged arm (*brachiolus*) that allows the reading of coordinates.

The orthographic projection of the instrument according to DE ROJAS allows a significant simplification of handling: Instead of a *Regula* with *Brachiolus*, the *Regula* is equipped with a cursor that can slide parallel in a track, whose markings can be directly used as declination, altitude, or latitude markings. However, due to the strong compression of the coordinate lines at the edge, there is a lack of accuracy there.

Image Distortions

On the *Saphæa*, due to the stereographic projection, areas near the edge are greatly stretched, while the central area appears compressed (“pincushion distortion”). On the other hand, in the model after DE ROJAS, the central area is enlarged, while the edge is extremely compressed (“barrel distortion”). DE LA HIRE sought a distortion-free projection so that the reading accuracy would be equally good over the entire surface (Greenwich 1976/1989; see also Section 8.3). However, the solution he found produces an instrument that, in use, again shows all the complications of the *Saphæa*.

8.1 Solution of Specific Tasks with the Universal Astrolabe

All models of universal astrolabes are excellently suited for the transformation of coordinates between two spherical grids. Usually, the *Regula* is used in connection with the hinged pointer (*brachiolus*) to set coordinates in one grid, then the *Regula* is rotated, and the coordinates in the other grid are read off.

In some tasks, an iterative process must be applied with the *Saphæa* and the model of DE LA HIRE, the avoidance of which is the main advantage of the model after DE ROJAS.

In this section, some typical applications of universal astrolabes are to be described. In operation, the *Saphæa* and the model of DE LA HIRE do not differ practically, which is why we want to distinguish in this section only between *Saphæa* and DE ROJAS.

Some of the descriptions may appear somewhat complicated. Due to the combination of several grids, the proper handling of these instruments actually requires some practice. Numerous related tasks can also be carried out with appropriate practice.

8.1.1 Sky and Horizon, Worldwide

If one sets the *Regula* to the geographical latitude φ indicated on the degree scale at the outer edge in the upper left or lower right quadrant, one can consider the position of the equatorial grid relative to the horizon. The horizon at the North Pole corresponds exactly to the celestial equator (*Regula* horizontal), the horizon at the Earth's equator is set by positioning the *Regula* vertically (it runs from pole to pole). On the *Regula*, one can read off the azimuths of rising and setting of objects with known declination at the intersection with the respective line for δ.

Only on the stereographic, angle-preserving *Saphæa* do we also clearly recognize that the daily paths of objects at the celestial equator cut the horizon at the steepest angle, while the paths of objects near the northern or southern horizon show more grazing intersections with the horizon.

8.1.2 Coordinate Transformation

In connection with the hinged pointer (*brachiolus*) and its pivotable support (*Regula*) or the cursor of the DE ROJAS instrument, the grid can be used very simply for transformation between ecliptic and equatorial coordinates.

8.1.2.1 Ecliptic to Equatorial

We set the *Regula* parallel to the equator of the grid. The entire grid now represents ecliptic coordinates.

Saphæa: We set the ecliptic coordinates using the *brachiolus*.

de Rojas: We set the ecliptic coordinates using the cursor.

Now we rotate the *Regula* so that it is parallel to the slanted ecliptic line. The *brachiolus* or cursor must not shift relative to the *Regula* during this! Now we interpret the grid as an equatorial grid and can conveniently read off the coordinates at the tip of the *brachiolus*/cursor, while paying attention to the hemisphere.

8.1.2.2 Equatorial to Ecliptic

The grid first represents equatorial coordinates. We set the *Regula* parallel to the ecliptic line in the grid.

Saphæa: We set the equatorial coordinates using the *brachiolus*.

de Rojas: We set the equatorial coordinates using the cursor.

Now we rotate the *Regula* so that it lies parallel to the equator of the grid. The *brachiolus* or cursor must not shift relative to the *Regula* during this! Now we interpret the grid as an ecliptic grid and can simply read off the coordinates at the tip of the *brachiolus*/cursor, while paying attention to the hemisphere.

8.1.3 Determination of the Equatorial Coordinates of the Sun

We take the ecliptic longitude of the Sun from the calendar circle. We orient the *Regula* parallel to the ecliptic. The *Regula* is labeled with ecliptic longitudes, so we can directly read off the equatorial coordinates from the grid at the point of the corresponding ecliptic longitude, where longitudes from 0° to 90° correspond to right ascensions $0^h \dots 6^h$, 90° … 270° correspond to $6^h \dots 18^h$, and 270° … 360° to $18^h \dots 24^h$.

By reversing the steps, we also find the generally two dates on which the Sun has a specific declination: we read off the two corresponding ecliptic longitudes from the *Regula* and look up the dates in the calendar circle.

8.1.4 Determination of the Sun's Noon Altitude for Any Location

In this simple operation, we again use the *Regula* as the local horizon line. On the outermost scale of the backside, we find a quadruple 90° division. The quadrants at the top left and bottom right are to be understood here again as latitude indications φ, the other two quadrants as polar distance $90 - \varphi$.

We align the *Regula* horizontally, thus setting the local horizon at the North Pole; the *brachiolus*/cursor points upwards if the Sun has northern declination; otherwise, we turn the *Regula* around. We set the tip of the *brachiolus*/cursor at the left edge of the grid to the Sun's declination δ, which we determined as in the previous section. Now we rotate the *Regula* with the *brachiolus*/cursor clockwise until the latitude φ is reached on the outer scale: the tip of the *brachiolus*/cursor now points to the noon altitude $(90° - \varphi + \delta)$. For latitudes south of the equator, one rotates further; caution is then required with the edge markings!

Just as well, one can set the *Regula* to the local latitude and the Sun's declination with the *brachiolus*/cursor at the right edge of the grid. Then one rotates the *Regula* to the equator line and reads off the noon altitude at the position of the *brachiolus*/cursor.

This task may be easier to do in one's head, but it can help to get used to the grids and reading possibilities!

8.1.5 Date on which the Sun Reaches a Specific Noon Altitude

Saphæa: With the *Regula* at the equator line, we set the tip of the *brachiolus* to the given noon altitude at the right edge of the grid. Now we rotate the *Regula* to the local latitude φ and read off the declination at the right edge with the *brachiolus*.

de Rojas: We set the *Regula* to the local latitude. Now we slide the cursor to the right until it intersects the right edge of the grid at the desired altitude. At the edge, we find the declination of the Sun.

From the declination, we determine the corresponding dates similarly to Section 8.1.3.

8.1.6 Sunrise and Sunset of the Sun, Twilight Times

We determine the declination of the Sun as in Section 8.1.3. Now we set the *Regula* to the local horizon (on the outermost scale, we set the geographic latitude φ). The declination arc of the Sun now represents its daily path. In the Indo-Arabic numerals along the meridians, we recognize an hour scale, from which we can directly read off the mean time of sunrise (upper scale) or sunset (lower scale) at the line that intersects the horizon at the current declination of the Sun.

The declination arc of the Sun above the horizon formed by the *Regula* also represents half the day arc, from which we can simply count the half length of the daylight (the hours until the 12 o'clock mark at the right edge of the grid).

The lower edge of the *Regula* (*Linea auroræ*) represents the -18° twilight line. Just as described above for the horizon line, we can read off the times of the beginning and end of astronomical twilight at the meridian arc of the intersection with the Sun's declination.

8.1.7 Position of the Sun at Any Time of Day

We determine the declination of the Sun as in Section 8.1.3. The *Regula* is set to the geographic latitude φ. The position of the Sun can now be set in the hour grid at its daily path (the declination line) using the *brachiolus* or cursor.

Saphæa: Now we rotate the *Regula* back to the equator line. The grid now represents azimuthal coordinates. At the position of the tip of the *brachiolus*, we can read off the solar altitude; we can bring down the meridians to the *Regula*, where we can read off the azimuth.

de Rojas: The altitude marking on the cursor directly gives the solar altitude. If we also want to know the azimuth, we also rotate the *Regula* to the equator line and read the azimuth at the intersection point of the corresponding great circle ellipse (which now represents the azimuth circle) with the equator line (here the horizon).

8.1.8 Time at Which the Sun Reaches a Specific Altitude

We determine the declination of the Sun as in Section 8.1.3. The *Regula* is set to the geographic latitude φ.

Saphæa: This task requires an iterative procedure [Saunders 1984, p.73].

1. We set the tip of the *brachiolus* to a position along the declination circle that, according to our estimation, should have the corresponding altitude above the horizon.
2. Then we rotate the *Regula* to the equator line and check whether the *brachiolus* lies on the desired altitude circle.

As long as the now displayed altitude does not correspond to the desired one, we try another position on the same declination line in step (1).

de Rojas: We slide the cursor until the desired altitude intersects the daily path of the Sun. At the hour marks of the great circle ellipses, we can read off the two times (morning/afternoon) at this intersection point.

When we rotate the *Regula* into the equator, we can read off the azimuths on the great circle arcs.

8.1.9 Solar Altitude and Time at Which the Sun Reaches a Specific Azimuth

This is the *Qiblah* problem from Section 7.7.7.3 (Section 7.7.7.3 is not available here), which can be solved very easily with planispheric astrolabes, but with universal astrolabes of all types can only be solved iteratively [Saunders 1984, p.75].

1. We determine the declination of the Sun as in Section 1.3.
2. The *Regula* is set to the observer's latitude φ.
3. The *brachiolus*/cursor is set with the declination line at an estimated plausible altitude.
4. We read the time on the hour lines.

Saphæa:

5. We rotate the *Regula* to the equator line and check whether the *brachiolus* points to the desired azimuth.
6. In this case, we see the altitude on the grid.

de Rojas:

5. On the cursor, we directly see the altitude.
6. We rotate the *Regula* to the equator line and check whether the cursor lies on the desired azimuth line.

Steps (2) to (6) are repeated until the *brachiolus*/cursor lies at the correct azimuth in the horizontal grid.

8.1.10 Culmination Altitude of a Star

This task is similar to Section 8.1.4. We rotate the *Regula* to the geographic latitude and set the declination of the star at the right edge of the grid.

Saphæa: Now we rotate the *Regula* to the equator line and recognize the culmination altitude at the edge.

de Rojas: The scale on the cursor gives the culmination altitude directly at the intersection with the edge line.

8.1.11 Determination of Latitude from Culmination Altitude Measurement

We set the *Regula* to the equator line and the *brachiolus*/cursor to the measured altitude of the celestial body (Sun or star). Then we rotate the *Regula* until the set point reaches the given declination. The position of the *Regula* now gives the geographic latitude φ, readable at the outer edge.

8.1.12 Rising and Setting of a Star

In this task, which is very simple with a planispheric astrolabe (Section 3.13, not available here), we cannot avoid calculations with universal astrolabes, as we always have to consider the Sun's position.

We determine the Sun's longitude $\lambda_\odot$ from the calendar circle and its right ascension $\alpha_\odot$ from the grid. With the star's position α_*, we determine the difference in right ascension $\Delta\alpha = (\alpha_* - \alpha_\odot)$ mod 24. The star culminates after this time.

We rotate the *Regula* to our geographic latitude φ and determine the hour angles $\widehat{H}_\uparrow, \widehat{H}_\downarrow$ of rising and setting on the declination line (daily path) of the star. *Note:* The hour angles of setting must be increased by 12 compared to the labeling!

The rising and setting times are now given by

$$t_\uparrow = (\widehat{H}_\uparrow + \Delta\alpha) \bmod 24^h \qquad \text{and} \qquad t_\downarrow = (\widehat{H}_\downarrow + \Delta\alpha) \bmod 24^h$$

The corresponding azimuths can be seen on the *Regula*.

8.1.13 Time at Which a Star Reaches a Specific Altitude

This problem is a generalization of the problem in Section 8.1.12 and would be much easier to solve with a planispheric astrolabe for the corresponding latitude (see Section 3.14, not available here).

1. We again determine the right ascension of the Sun $\alpha_\odot$, of the star α_*, and $\Delta\alpha = (\alpha_* - \alpha_\odot)$ mod 24.
2. The *Regula* is set to the geographic latitude φ.

Saphæa:

3. (a) Along the declination line of the star, we estimate a position that could correspond to the desired altitude h above the horizon and set the *brachiolus* to this position.

   (b) We rotate the *Regula* to the equator line and check our assumption in the now present azimuthal grid.

Steps (2) to (3b) are repeated until the *brachiolus* in step (3b) lies at altitude h.

de Rojas:

3. We bring the desired altitude marking h on the *cursor* onto the declination line of the star.

4. With the *Regula* at the geographic latitude φ, we read the two hour angles $(\widehat{H}_\nearrow, \widehat{H}_\searrow)$ of the star at which it passes the desired altitude h. *Note:* The hour angles after culmination must be increased by 12 compared to the labeling!
5. The desired times are given by

$$t_\nearrow = (\widehat{H}_\nearrow + \Delta\alpha) \bmod 24^h \qquad \text{and} \qquad t_\searrow = (\widehat{H}_\searrow + \Delta\alpha) \bmod 24^h$$

6. With the *Regula* at the equator line, we can read off the corresponding azimuths of the star in the grid.

8.1.14 Star Position on a Given Date

One of the simplest applications for the planispheric astrolabe for the corresponding latitude (see Section 3.12, not available here) again requires calculations here.

We again determine the right ascension of the Sun $\alpha_{\odot}$, of the star α_*, and $\Delta\alpha = (\alpha_* - \alpha_{\odot})$ mod 24. The hour angle of the star is given by $\widehat{H}_* = t_{\odot} - \Delta\alpha$ mod 24. *Note:* Compared to the modern concept of hour angle H, this is also shifted by 12 hours $\widehat{H} < 12$ is therefore a position in the eastern sky.

The *Regula* is set to the geographic latitude φ.

Saphæa: The tip of the *brachiolus* is set at the intersection point of the daily path of the star (declination δ) with the hour angle.

de Rojas: The cursor is set at the intersection point of the daily path of the star (declination δ) with the hour angle. The altitude is readable on the cursor scale.

After rotating the *Regula* back, azimuth and altitude can be read off in the horizontal grid.

Literature

- National Maritime Museum Greenwich (ed.): *The Planispheric Astrolabe*, London 1976, 2nd edition 1989.
- Harold Saunders: *All the Astrolabes*, Oxford 1974.
- Ernst Zinner: *German and Dutch Astronomical Instruments of the 11th to 18th Centuries*, 2nd edition 1967.